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ROBERT DUBIL

An Arbitrage Guide to Financial Markets

Robert Dubil



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To Britt, Elsa, and Ethan

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The Purpose and Structure of Financial Markets

1.1 OVERVIEW

Financial markets play a major role in allocating wealth and excess savings to productive ventures in the global economy. This extremely desirable process takes on various forms. Commercial banks solicit depositors' funds in order to lend them out to businesses that invest in manufacturing and services or to home buyers who finance new construction or redevelopment. Investment banks bring to market offerings of equity and debt from newly formed or expanding corporations. Governments issue short- and long-term bonds to finance construction of new roads, schools, and transportation networks. Investors—bank depositors and securities buyers—supply their funds in order to shift their consumption into the future by earning interest, dividends, and capital gains.

The process of transferring savings into investment involves various market participants: individuals, pension and mutual funds, banks, governments, insurance companies, industrial corporations, stock exchanges, over-the-counter dealer networks, and others. All these agents can at different times serve as demanders and suppliers of funds, and as transfer facilitators.

Economic theorists design optimal securities and institutions to make the process of transferring savings into investment most efficient. “Efficient” means to produce the best outcomes—lowest cost, least disputes, fastest, etc.—from the perspective of security issuers and investors, as well as for society as a whole. We start this book by addressing briefly some fundamental questions about today's financial markets. Why do we have things like stocks, bonds, or mortgage-backed securities? Are they outcomes of optimal design or happenstance? Do we really need “greedy” investment bankers, securities dealers, or brokers soliciting us by phone to purchase unit trusts or mutual funds? What role do financial exchanges play in today's economy? Why do developing nations strive to establish stock exchanges even though often they do not have any stocks to trade on them?

Once we have basic answers to these questions, it will not be difficult to see why almost all the financial markets are organically the same. Like automobiles made by Toyota and Volkswagen which all have an engine, four wheels, a radiator, a steering wheel, etc., all interacting in a predetermined way, all markets, whether for stocks, bonds, commodities, currencies, or any other claims to purchasing power, are built from the same basic elements.

All markets have two separate segments: original-issue and resale. These are characterized by different buyers and sellers, and different intermediaries. They perform different timing functions. The first transfers capital from the suppliers of funds (investors) to the demanders of capital (businesses). The second transfers

capital from the suppliers of capital (investors) to other suppliers of capital (investors). The original-issue and resale segments are formally referred to as:

- *Primary markets* (issuer-to-investor transactions with investment banks as intermediaries in the securities markets, and banks, insurance companies, and others in the loan markets).
- *Secondary markets* (investor-to-investor transactions with broker-dealers and exchanges as intermediaries in the securities markets, and mostly banks in the loan markets).

Secondary markets play a critical role in allowing investors in the primary markets to transfer the risks of their investments to other market participants.

All markets have the originators, or issuers, of the claims traded in them (the original demanders of funds) and two distinctive groups of agents operating as investors, or suppliers of funds. The two groups of funds suppliers have completely divergent motives. The first group aims to eliminate any undesirable risks of the traded assets and earn money on repackaging risks, the other actively seeks to take on those risks in exchange for uncertain compensation. The two groups are:

- *Hedgers* (dealers who aim to offset primary risks, be left with short-term or secondary risks, and earn spread from dealing).
- *Speculators* (investors who hold positions for longer periods without simultaneously holding positions that offset primary risks).

The claims traded in all financial markets can be delivered in three ways. The first is an immediate exchange of an asset for cash. The second is an agreement on the price to be paid with the exchange taking place at a predetermined time in the future. The last is a delivery in the future contingent on an outcome of a financial event (e.g., level of stock price or interest rate), with a fee paid upfront for the right of delivery. The three market segments based on the delivery type are:

- *Spot or cash markets* (immediate delivery).
- *Forwards markets* (mandatory future delivery or settlement).
- *Options markets* (contingent future delivery or settlement).

We focus on these structural distinctions to bring out the fact that all markets not only transfer funds from suppliers to users, but also risk from users to suppliers. They allow *risk transfer* or *risk sharing* between investors. The majority of the trading activity in today's market is motivated by risk transfer with the acquirer of risk receiving some form of sure or contingent compensation. The relative price of risk in the market is governed by a web of relatively simple arbitrage relationships that link all the markets. These allow market participants to assess instantaneously the relative attractiveness of various investments within each market segment or across all of them. Understanding these relationships is mandatory for anyone trying to make sense of the vast and complex web of today's markets.

1.2 RISK SHARING

All financial contracts, whether in the form of securities or not, can be viewed as bundles, or packages of unit payoff claims (mini-contracts), each for a specific date in the future and a specific set of outcomes. In financial economics, these are referred to as *state-contingent claims*.

Let us start with the simplest illustration: an insurance contract. A 1-year life insurance policy promising to pay \$1,000,000 in the event of the insured's death can be viewed as a package of 365 daily claims (lottery tickets), each paying \$1,000,000 if the holder dies on that day. The value of the policy upfront (the premium) is equal to the sum of the values of all the individual tickets. As the holder of the policy goes through the year, he can discard tickets that did not pay off, and the value of the policy to him diminishes until it reaches zero at the end of the coverage period.

Let us apply the concept of state-contingent claims to known securities. Suppose you buy one share of XYZ SA stock currently trading at €45 per share. You intend to hold the share for 2 years. To simplify things, we assume that the stock trades in increments of €0.05 (tick size). The minimum price is €0.00 (a limited liability company cannot have a negative value) and the maximum price is €500.00. The share of XYZ SA can be viewed as a package of claims. Each claim represents a contingent cash flow from selling the share for a particular price at a particular date and time in the future. We can arrange the potential price levels from €0.00 to €500.00 in increments of €0.05 to have overall 10,001 price levels. We arrange the dates from today to 2 years from today (our holding horizon). Overall we have 730 dates. The stock is equivalent to $10,001 \times 730$, or 7,300,730 claims. The easiest way to imagine this set of claims is as a rectangular chessboard where on the horizontal axis we have time and on the vertical axis the potential values the stock can take on (states of nature). The price of the stock today is equal to the sum of the values of all the claims (i.e., all the squares of the chessboard).

Table 1.1 Stock held for 2 years as a chessboard of contingent claims in two dimensions: time (days 1 through 730) and prices (0.00 through 500.00)

| | | | | | | | | |
|----------------|--------|--------|--------|--------|--------|--------|--------|--------|
| 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 |
| 499.95 | 499.95 | 499.95 | 499.95 | 499.95 | 499.95 | 499.95 | 499.95 | 499.95 |
| 499.90 | 499.90 | 499.90 | 499.90 | 499.90 | 499.90 | 499.90 | 499.90 | 499.90 |
| 499.85 | 499.85 | 499.85 | 499.85 | 499.85 | 499.85 | 499.85 | 499.85 | 499.85 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 60.35 | 60.35 | 60.35 | 60.35 | 60.35 | 60.35 | 60.35 | 60.35 | 60.35 |
| 60.30 | 60.30 | 60.30 | 60.30 | 60.30 | 60.30 | 60.30 | 60.30 | 60.30 |
| 60.25 | 60.25 | 60.25 | 60.25 | 60.25 | 60.25 | 60.25 | 60.25 | 60.25 |
| 60.20 | 60.20 | 60.20 | 60.20 | 60.20 | 60.20 | 60.20 | 60.20 | 60.20 |
| 60.15 | 60.15 | 60.15 | 60.15 | 60.15 | 60.15 | 60.15 | 60.15 | 60.15 |
| 60.10 | 60.10 | 60.10 | 60.10 | 60.10 | 60.10 | 60.10 | 60.10 | 60.10 |
| 60.05 | 60.05 | 60.05 | 60.05 | 60.05 | 60.05 | 60.05 | 60.05 | 60.05 |
| 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 |
| 59.95 | 59.95 | 59.95 | 59.95 | 59.95 | 59.95 | 59.95 | 59.95 | 59.95 |
| 59.90 | 59.90 | 59.90 | 59.90 | 59.90 | 59.90 | 59.90 | 59.90 | 59.90 |
| 59.85 | 59.85 | 59.85 | 59.85 | 59.85 | 59.85 | 59.85 | 59.85 | 59.85 |
| 59.80 | 59.80 | 59.80 | 59.80 | 59.80 | 59.80 | 59.80 | 59.80 | 59.80 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 0.45 | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 |
| 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 |
| 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 |
| 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | 2 | ... | 364 | 365 | 366 | ... | 729 | 730 |
| Days | | | | | | | | |
| Stock price \$ | | | | | | | | |

A forward contract on XYZ SA’s stock can be viewed as a subset of this rectangle. Suppose we enter into a contract today to purchase the stock 1 year from today for €60. We intend to hold the stock for 1 year after that. The forward can be viewed as $10,001 \times 365$ rectangle with the first 365 days’ worth of claims taken out (i.e., we are left with the latter 365 columns of the board, the first 365 are taken out). The cash flow of each claim is equal to the difference between the stock price for that state of nature and the contract price of €60. A forward carries an obligation on both sides of the contract so some claims will have a positive value (stock is above €60) and some negative (stock is below €60).

Table 1.2 One-year forward buy at €60 of stock as a chessboard of contingent claims. Payoff in cells is equal to $S - 60$ for year 2. No payoff in year 1

| | | | | | | | | |
|------|------|------|------|--------|--------|--------|--------|-----|
| 0.00 | 0.00 | 0.00 | 0.00 | 440.00 | 440.00 | 440.00 | 500.00 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 439.95 | 439.95 | 439.95 | 499.95 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 439.90 | 439.90 | 439.90 | 499.90 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 439.85 | 439.85 | 439.85 | 499.85 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.35 | 0.35 | 60.35 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.30 | 0.30 | 60.30 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.25 | 0.25 | 60.25 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.20 | 0.20 | 60.20 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.15 | 0.15 | 60.15 | |
| 0.00 | 0.00 | ... | 0.00 | 0.10 | ... | 0.10 | 60.10 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.05 | 60.05 | |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 60.00 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -0.05 | -0.05 | -0.05 | 59.95 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -0.10 | -0.10 | -0.10 | 59.90 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | -0.15 | -0.15 | 59.85 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -0.20 | -0.20 | -0.20 | 59.80 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.55 | -59.55 | -59.55 | 0.45 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.60 | -59.60 | -59.60 | 0.40 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.65 | -59.65 | -59.65 | 0.35 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.70 | ... | -59.70 | 0.30 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.75 | -59.75 | -59.75 | 0.25 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.80 | -59.80 | -59.80 | 0.20 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.85 | -59.85 | -59.85 | 0.15 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.90 | -59.90 | -59.90 | 0.10 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -59.95 | -59.95 | -59.95 | 0.05 | |
| 0.00 | 0.00 | 0.00 | 0.00 | -60.00 | -60.00 | -60.00 | 0.00 | |
| 1 | 2 | ... | 364 | 365 | 366 | ... | 729 | 730 |

Days

Stock price S

An American call option contract to buy XYZ SA’s shares for €60 with an expiry 2 years from today (exercised only if the stock is above €60) can be represented as a $8,800 \times 730$ subset of our original rectangular $10,001 \times 730$ chessboard. This time, the squares corresponding to the stock prices of €60 or below are eliminated, because they have no value. The payoff of each claim is equal to the intrinsic (exercise) value of the call. As we will see later, the price of each claim today is equal to at least that.

Table 1.3 American call struck at €60 as a chessboard of contingent claims. Expiry 2 years. Payoff in cells is equal to $S - 60$ if $S > 60$

| | | | | | | | | | |
|-------------|----------|-----|------------|------------|------------|-----|------------|------------|--------------------|
| 440.00 | 440.00 | | 440.00 | 440.00 | 440.00 | | 440.00 | 440.00 | 500.00 |
| 439.95 | 439.95 | | 439.95 | 439.95 | 439.95 | | 439.95 | 439.95 | 499.95 |
| 439.90 | 439.90 | | 439.90 | 439.90 | 439.90 | | 439.90 | 439.90 | 499.90 |
| 439.85 | 439.85 | | 439.85 | 439.85 | 439.85 | | 439.85 | 439.85 | 499.85 |
| 0.35 | 0.35 | | 0.35 | 0.35 | 0.35 | | 0.35 | 0.35 | 60.35 |
| 0.30 | 0.30 | | 0.30 | 0.30 | 0.30 | | 0.30 | 0.30 | 60.30 |
| 0.25 | 0.25 | | 0.25 | 0.25 | 0.25 | | 0.25 | 0.25 | 60.25 |
| 0.20 | 0.20 | | 0.20 | 0.20 | 0.20 | | 0.20 | 0.20 | 60.20 |
| 0.15 | 0.15 | | 0.15 | 0.15 | 0.15 | | 0.15 | 0.15 | 60.15 |
| 0.10 | 0.10 | ... | 0.10 | 0.10 | 0.10 | ... | 0.10 | 0.10 | 60.10 |
| 0.05 | 0.05 | ... | 0.05 | 0.05 | 0.05 | ... | 0.05 | 0.05 | 60.05 <i>Stock</i> |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 60.00 <i>Price</i> |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 59.95 <i>S</i> |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 59.90 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 59.85 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 59.80 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.45 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.40 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | ... | 0.00 | 0.00 | 0.35 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.30 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.25 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.20 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.15 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.10 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.05 |
| 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 | | 0.00 | 0.00 | 0.00 |
| <i>1</i> | <i>2</i> | ... | <i>364</i> | <i>365</i> | <i>366</i> | ... | <i>729</i> | <i>730</i> | |
| <i>Days</i> | | | | | | | | | |